1. (a) Let $\mathbf{a} = \overrightarrow{BA} = \langle 1, 1, 1\rangle$ and $\mathbf{b} = \overrightarrow{BC} = \langle -1, 0, -2\rangle$. Then

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{-3}{\sqrt{3}\sqrt{5}} = -\frac{3}{\sqrt{15}}$$

Therefore $\theta = 2.46$ in radians, or $\theta = 141°$.

(b) The plane has normal vector $\langle 3, -1, -5\rangle$ and goes through $(1, 1, 1)$, so the equation is $3(x-1) - 1(y-1) - 5(z-1) = 0$ or $3x - y - 5z + 3 = 0$.

(c) Let $\mathbf{a} = \langle 1, -4, 2\rangle$ and $\mathbf{b} = \langle 1, -1, 0\rangle$ be the normal vectors of these planes. Then the direction vector of the line is $\langle 2, 2, 3\rangle$. Since the lines passes through the origin, the equation for the line is $\mathbf{r}(t) = \langle 2t, 2t, 3t\rangle$.

2. (a) At $(0, 0)$, we have $t = 0$. We have $\dfrac{dy}{dx} = \dfrac{5\cos t}{2t+1}$, and when $t = 0$, this is 5.

(b) From part (a), we know that the curve passes through $(0, 0)$ and has slope 5 there. The graph on the left is the only one with these properties.

3. (a) f_x is positive in the right half of the graph. f_y is negative on the bottom half of the graph. So anywhere in the bottom right part of the graph.

(b) Any point centered horizontally on the graph, where the level curves are horizontal.

4. (a) $\mathbf{a}(t) = \mathbf{r}''(t) = \langle -\cos t, -\sin t, -9.8\rangle$.

(b) The velocity of the particle is $\mathbf{r}'(t) = \langle -\sin t, \cos t, -9.8t\rangle$, and the speed is $\sqrt{\sin^2 t + \cos^2 t + (9.8t)^2} = \sqrt{1 + (9.8t)^2}$. The particle hits the ground when $z = 9.8 - 4.9t^2 = 0$, which is when $t = \sqrt{2}$. At this time, the speed is $\sqrt{1 + (9.8)^2(2)} = \sqrt{193.08} = 13.9$ m/s

5. (a) We have $\mathbf{r}'(t) = \langle 1, 2t, 0\rangle$ and $\mathbf{r}''(t) = \langle 0, 2, 0\rangle$. Then the curvature at $t = 1$ is

$$\frac{|\mathbf{r}'(1)\times\mathbf{r}''(1)|}{|\mathbf{r}'(1)|^3} = \frac{|\langle 0, 0, 2\rangle|}{|\langle 1, 2, 0\rangle|^3} = \frac{2}{5^{3/2}} = \frac{2\sqrt{5}}{25}.$$

(b) The vector $\mathbf{r}'(1) = \langle 1, 2, 0\rangle$ is tangent to the curve at the point $\mathbf{r}(1) = \langle 1, 0, 1\rangle$. So the normal plane is $1(x-1) + 2(y-0) + 0(z-1) = 0$, or $x + 2y - 1 = 0$.

6. (a) First we compute $\mathbf{r}'(t) = \langle 4t, -4t, 2t\rangle$ and $|\mathbf{r}'(t)| = 6|t|$. Since we only care about positive t, this is equal to $6t$. The arc length function is $s(t) = \displaystyle\int_0^t 6u\,du = 3t^2$. This means that $t^2 = s/3$ and $t = \sqrt{s/3}$. Substituting this expression in for t, we get $\mathbf{r}(s) = \left\langle \dfrac{2s}{3}, 1 - \dfrac{2s}{3}, 5 + \dfrac{s}{3}\right\rangle$.

(b) The length of the curve from 0 to 3 is $s(3) = 3(3^2) = 27$.