

Print your name: _____

Score

1. Determine $\int_C (x + y) ds$ if C is the curve consisting of the line from $(0, 0)$ to $(2, 2)$ followed by the half circle that goes counterclockwise from $(2, 2)$ to $(-2, -2)$.

Solution: We'll split the curve in two. The line segment can be parametrized by $\mathbf{r}(t) = \langle 2t, 2t \rangle$, $0 \leq t \leq 1$. Then the integral is

$$\int_0^1 4t\sqrt{8} dt = \int_0^1 8\sqrt{2}t dt = 4\sqrt{2}t^2 \Big|_0^1 = 4\sqrt{2}.$$

The circle has radius $2\sqrt{2}$, so can be parametrized by $\mathbf{r}(t) = 2\sqrt{2}\langle \cos t, \sin t \rangle$, $\pi/4 \leq t \leq 5\pi/4$. So the integral is

$$\int_{\pi/4}^{5\pi/4} 2\sqrt{2}(\cos t + \sin t) 2\sqrt{2} dt = 8(\sin t - \cos t) \Big|_{\pi/4}^{5\pi/4} = 0.$$

Then the final answer is $4\sqrt{2}$.

