Miscellaneous notes on parametric equations

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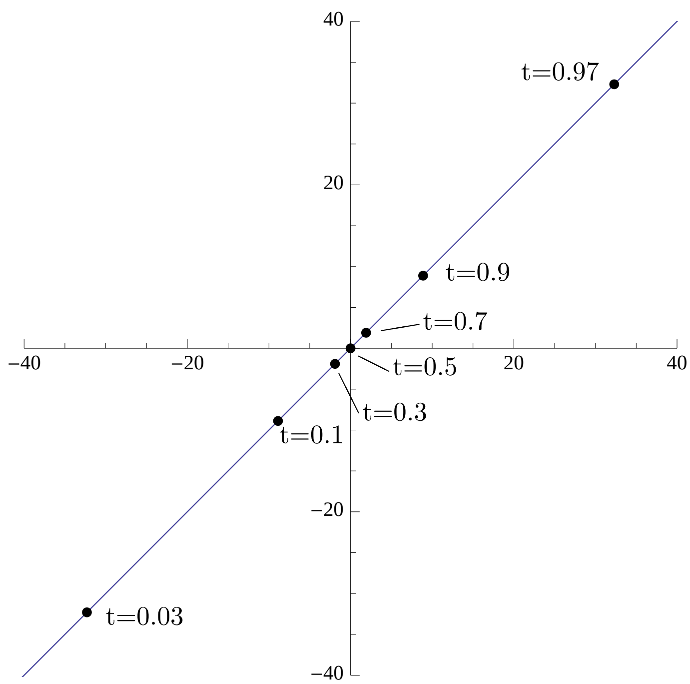
An entire line parametrized by a finite interval

It is possible for a parametric curve to extend off to infinity even though the parameter is bounded. For example, the following parametric equations

$$x = \frac{1}{1-t} - \frac{1}{t}$$
$$y = \frac{1}{1-t} - \frac{1}{t}$$

trace out the line $x = y$ as t goes from 0 to 1. Here is a graph of the line with some t-values labelled.



Area under semicircle using parametric equations

The upper half of a circle is parametrized by

$$x = \cos t$$
$$y = \sin t$$

as t goes from 0 to π. The area under the semicircle is given by

$$\begin{aligned}
\int_{-1}^{1} y\,dx &= \int_{\pi}^{0} \sin t(-\sin t)\,dt \\
&= \int_{0}^{\pi} \sin^2 t\,dt \\
&= \frac{1}{2}\int_{0}^{\pi} (1 - \cos 2t)dt \\
&= \frac{1}{2}\left(t - \frac{1}{2}\sin t\right)\Bigg|_{0}^{\pi} \\
&= \frac{1}{2}(\pi) = \boxed{\frac{\pi}{2}}
\end{aligned}$$

Notice that the integral originally goes from π to 0 because $t = \pi$ when $x = -1$ and $t = 0$ when $x = 1$. Then when we switch 0 and π we multiply by -1, which cancels out the minus sign already in the integrand. We integrate $\sin^2 t$ using the trig identity $\sin^2 t = \frac{1}{2}(1 - \cos t)$.