Analyzing an oscillation

We often want to take a solution to a differential equation

$$c_1 \cos(\omega_0 t) + c_2 \sin(\omega_o t) \qquad \text{(nonstandard form)}$$

and figure out the amplitude, frequency, period, and phase of the oscillation. To do this, it is best to rewrite the function as a single sine or cosine. We will use a cosine:

$$R \cos(\omega_0 t - \delta) \qquad \text{(standard form)}$$

Once we do this, we see that the amplitude, which is the distance from the center (neutral position) to the place where the spring is stretched the most, is R. The frequency is ω_0, and is measured in radians per second. The period is $2\pi/\omega$ and tells you how many seconds it takes to complete one oscillation. The phase is δ, which tells you which part of the cycle the spring is in at time $t = 0$.

By natural length, I mean the length that the spring would be if the mass were attached but at rest. Since ω_0 is the same in both the nonstandard and standard forms, we really only need to know how to find R and δ. We use

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad \text{(cosine addition formula)}$$

to get

$$R\cos(\omega_0 t - \delta) = R\cos\delta\cos(\omega_0 t) + R\sin\delta\sin(\omega_0 t)$$

So, we have

$$c_1 = R\cos\delta \qquad\qquad c_2 = R\sin\delta$$

This means $c_1^2 + c_2^2 = R^2$, so we can find R from c_1 and c_2. Also, $c_2/c_1 = \tan(\delta)$, so we can find δ by carefully taking an inverse tangent.