1a. $ce^{(1/5)t} - \dfrac{1}{26}e^{-5t}$

1b. $ce^{-1/t}$

2. The differential equation is $Q' = 10r - \dfrac{Q}{50}r$, where r is the rate of water flow, and Q is the quantity of dye. The initial condition is $Q(0) = 0$. The solution is $Q(t) = 500 - 500e^{-rt/50}$. Using the condition that $Q(10) = 250$, we get $r = 5\ln 2 \approx 3.47$.

3. $y'' + 2y' + 10y = 0$: E because it describes an underdamped spring.
$y'' + 2y' + y = 0$: A because it describes a critically damped spring.
$y'' + 64y = 0$: C because it describes a spring with no damping.
$y'' + 5y' - 6y = 0$: B because its general solution is $c_1e^{3t} + c_2e^{2t}$, which grows exponentially.
$y'' + 64y = 4\sin(8t)$: D because it describes an undamped spring with resonance.
$y' + y = 3$: F because it has a single stable equilibrium (at $y = 3$).
$y' = y - y^2$: G because it has an unstable equilibrium and a stable equilibrium.

4. The equation is $u'' + 2u' + 2u = 2\cos(\sqrt{2}\,t), u(0) = 0, u'(0) = 0$. The solution to the homogeneous version of the equation is $y_h = e^{-t}(c_1\cos t + c_2\sin t)$. The particular solution is of the form $A\cos(\sqrt{2}\,t) + B\sin(\sqrt{2}\,t)$, and using the method of undetermined coefficients we find that the particular solution is $Y = \dfrac{1}{\sqrt{2}}\sin(\sqrt{2}\,t)$. Now we use our initial conditions to get c_1 and c_2, which gives a final answer of $u(t) = -e^{-t}\sin t + \dfrac{1}{\sqrt{2}}\sin(\sqrt{2}\,t)$.

   The first term is the transient part and the second term is the steady state part. You could also have solved this problem using Laplace transforms.

5. $c_1\sqrt{t} + c_2t$

6a. $\dfrac{5e^{-2\pi s}e^{2\pi}}{(s-1)^2 + 25}$ and $e^{-s}\left(\dfrac{2}{s^2} + \dfrac{2}{s} + \dfrac{1}{s} + \dfrac{1}{s^2+1}\right)$

6b. $e^{-3t/2}\left[\cos\left(\dfrac{\sqrt{3}}{2}t\right) - \sqrt{3}\sin\left(\dfrac{\sqrt{3}}{2}t\right)\right]$ and $-u_\pi(t)\left(\cos 3t + \dfrac{1}{3}\sin 3t\right)$

7. The Laplace transform of the equation is $(s^2 + 5s + 6)Y = \dfrac{e^{-5s}}{s+1} + \dfrac{e^{-9s}e^{-4}}{s+1} + s + 2$. Solving for Y, we get $Y = e^{-5s}F(s) + e^{-4}e^{-9s}F(s) + \dfrac{1}{s+3}$, where

$$F(s) = \frac{1}{(s+1)(s+2)(s+3)} = \frac{1/2}{s+1} - \frac{1}{s+2} + \frac{1/2}{s+3}.$$

   The inverse Laplace transform of $F(s)$ is $f(t) = \frac{1}{2}e^{-t} - e^{-2t} + \frac{1}{2}e^{-3t}$. So the inverse Laplace transform of Y, and the answer to the problem, is

$$y(t) = e^{-3t} + u_5(t)f(t-5) + e^{-4}u_9(t)f(t-9).$$