## Math 307i

Print Your Name

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	15	
5	9	
6	20	
7	20	
Total	100	

## You should:

- write complete solutions or you may not receive credit.
- box your final answer.
- check that your exam contains a total of 9 pages.

## You may:

- use ten sheets of notes and a calculator.
- write on the backs of the pages if you need more room.

## Please do not:

- come to the front of the room to ask questions (raise your hand).
- share notes or calculators.
- use any electronic device other than a calculator.

**Signature.** Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: \_\_\_\_\_

- 1. (12 points)
  - (a) Find the general solution to the differential equation  $5y' = e^{-5t} + y$ .

(b) Find the general solution to the differential equation  $t^2y' = y$ , t > 0.

2. (12 points) A 50-liter tank is completely full of purified water. Water with a concentration of 10 milligrams per liter of purple dye flows into the tank at an unknown rate and the mixture flows out at the same rate. If the tank has a dye concentration of 5 milligrams per liter after 10 minutes, determine the rate at which the water flows into the tank.

3. (12 points) Match each differential equation with a graph of the solution. Mark your answers in the table.

	Differential equation   Gra	aph Differe	ntial equation	Graph	
	y'' + 2y' + 10y = 0	y'' + 2y	y' + y = 0		
	y'' + 64y = 0	y'' + 5y	y' - 6y = 0		
	$y'' + 64y = 4\sin(8t)$	y' + y =	= 3		
	$y' = y - y^2$				
Α		В		/	
С		D			$\land \land$
		$\forall$	$\frown \frown$		
Е		F			V
G					

- 4. (15 points) An object of mass 1 kg hangs from a spring with spring constant 2 N/m. A viscous damper with damping coefficient 2 N  $\cdot$  s/m is attached to the system. The mass is initially at rest in the equilibrium position, and an external force of  $2\cos(\sqrt{2}t)$  is applied to the mass.
  - (a) Determine the function that describes the position of the mass at any time.
  - (b) Label the steady state and transient parts of your answer. DON'T FORGET THIS PART.

5. (9 points) Use reduction of order to find the general solution to  $2t^2y'' - ty' + y = 0$ , given that  $y_1 = t$  is a solution.

- 6. (20 points)
  - (a) Find the Laplace transform of these two functions. i.  $u_{2\pi}(t)e^t \sin 5t$

ii.  $u_1(t) [t^2 + \sin(t-1)]$ 

(b) Find the inverse Laplace transform of these two functions.

i. 
$$\frac{s}{s^2 + 3s + 3}$$

ii. 
$$\frac{e^{-\pi s}(s+1)}{s^2+9}$$

7. (20 points) Solve the initial value problem

$$y'' + 5y' + 6y = \begin{cases} 0 & \text{if } 0 \le t < 5\\ e^{-(t-5)} & \text{if } 5 \le t < 9\\ 0 & \text{if } 9 \le t \end{cases}$$

with initial conditions y(0) = 1, y'(0) = -3.

If you are finished, congratulations. If you need more scratch paper for that last problem, use this page.