Resonance and Beats

Suppose you have an object of mass 1 kg hanging from a spring with damping coefficient 1/10 N/(m/s) and spring coefficient 1 N/m. If the mass is oscillating, what is the pseudo frequency (in radians per second)?

Answer $\sqrt{3.99}/2 \approx 0.999$

Now suppose there is a forcing function $R\sin(t)$. Solve the differential equation and determine the amplitude of the steady state response.

Answer The steady state solution is $-10R\cos(t)$, which has amplitutde $10R$.

What if you change the forcing function to $R\sin(2t)$. Now what is the amplitude of the steady state response?

Answer The steady state solution is $\frac{5R}{226}(\cos t + 25\sin t)$, which has amplitude $\frac{5\sqrt{226}}{226}R \approx 0.33R$.

Now suppose you have an object of mass 1 kg hanging from a spring with spring coefficient 1 N/m, without any damping or forcing function. What is the frequency of oscillation (in radians per second)?

Answer 1

Suppose there is a forcing function of $\cos(11t/10)$, and at time $t = 0$, the mass is at $u = 0$ with $u' = 0$. Determine the position of the mass at any time.

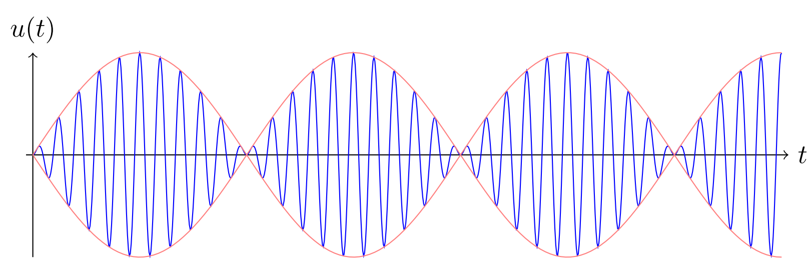
Answer $y = -\frac{100}{21}(\cos(\frac{11}{10}t) - \cos t)$.

Use the trig identity

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

to rewrite your answer as a product of two sin functions. Sketch a graph. This is an example of a *beat*. A similar thing happens when you play two tones at very close frequencies; you can then hear the amplitude of the sound increase and decrease in a periodic way.

Answer $u(t) = \frac{200}{21}\sin(\frac{21}{20}t)\sin(\frac{1}{20}t)$



The red line is the graph of $\pm\frac{200}{21}\sin(\frac{1}{20}t)$. The graph of u oscillates between these functions.