ESSAY PROBLEM, HOMEWORK 7

EIGENVALUES OF A 2×2 MATRIX WITH DETERMINANT 1

The **trace** of a matrix is the sum of the entries along the main diagonal. For example, the trace of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $a + d$.

Prove that if A is a 2×2 matrix with $\det(A) = 1$ and $\text{trace}(A) > 2$, then A has two distinct real eigenvalues.

Bonus problem (optional): If A is any 2×2 matrix, give conditions on the determinant and trace of A that distinguish when A has real and distinct eigenvalues, a single real eigenvalue, or complex eigenvalues.