

ESSAY PROBLEM, HOMEWORK 5

NULLSPACE VS RANGE

The purpose of this worksheet is to help you think about the differences between the nullspace of a matrix and the range of a matrix. First of all, you should know that **every subspace of R^n is the range of some matrix A and is also the nullspace of some (different) matrix B .**

Consider the subspace

$$W = \text{Sp} \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}.$$

1. Find a matrix A such that $W = \text{columnspace}(A) = \text{range}(A)$.
2. Find a matrix B such that $W = \text{nullspace}(B)$. [Hint: Use the method of Example 4 on page 182 to find the range of the matrix you found in Part 1. This should give you a homogeneous system of equations describing W . So the coefficient matrix of the homogeneous system is the matrix B you are looking for.]
3. Check your work on Part 2 by making sure that

$$B \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \vec{0} \text{ and } B \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \vec{0}.$$

4. Remember that a circle with radius 1 can be described as the solutions to the equation $x^2 + y^2 = 1$, or by the parametric equations $x = \cos(\theta), y = \sin(\theta)$. Which description of W (as the range of A or the image of B) is like giving parametric equations? Explain why.
5. (Optional, not graded) The subspace

$$\text{Sp} \left\{ \begin{bmatrix} 1 \\ a \\ c \end{bmatrix} \right\}$$

of R^3 is a line through the origin. Use the methods above to find a matrix C so that this subspace is the nullspace of C . Now you know what the “equations for a line” in R^3 look like. Why does it take two equations to define a line in R^3 and only one equation to define a line in R^2 ?