1. (a) The augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 5 & -2 \\ 1 & 3 & 6 & -2 \end{array}\right]$ row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & -3/2 & 1 \\ 0 & 1 & 5/2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$. So the solution is

$$x_1 = \frac{3}{2}x_3 + 1, x_2 = -\frac{5}{2}x_3 - 1, x_3 = x_3.$$

(b) No. Since the system of linear equations above has more than one solution, A must be singular, which means its columns are linearly dependent.

2. (a) The matrix $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 4 & 5 & 8 & 0 & 0 & 1 \end{array}\right]$ row reduces to $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 & -3/2 & 1/2 \end{array}\right]$, so

$$A^{-1} = \begin{bmatrix} -2 & 1/2 & 1/2 \\ 0 & 2 & -1 \\ 1 & -3/2 & 1/2 \end{bmatrix}.$$

(b) $(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 0 & 1 \\ 1/2 & 2 & -3/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}$

3. (a) There are many ways to find the determinant of A. The easiest is to do cofactor expansion down the first column, where there is only one nonzero entry. Remember that it is the $1, 2$ entry, so we need to put in a negative sign:

$$\det(A) = -1 \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 5 \\ 0 & 2 & 5 \end{vmatrix} = -1\big[(30+8) - (15+20)\big] = -3.$$

4. (a) True. If A is nonsingular, it has an inverse. Since $AA = A$, then $A^{-1}AA = A^{-1}A$. This simplifies to $A = I$.

(b) True. If A and B are symmetric then $A^T = A$ and $B^T = B$, and then $(A+B)^T = A^T + B^T = A + B$, so $A + B$ is symmetric.

(c) False. For example, if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, then A and B are both nonsingular, but $A + B$ is zero, which is singular.

5. Since $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly dependent set, there are scalars a_1, a_2, and a_3 which are not all zero, such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{0}$. Then there are scalars b_1, b_2, b_3, b_4 such that $b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + b_3\mathbf{v}_3 + b_4\mathbf{v}_4 = \mathbf{0}$, namely $b_1 = a_1, b_2 = a_2, b_3 = a_3, b_4 = 0$. The are not all zero since one of a_1, a_2, a_3 is nonzero. So the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.