Print Your Name

Student ID #

Problem	Total Points	Score
1	11	
2	11	
3	11	
4	9	
5	8	
Total	50	

Directions

- Please check that your exam contains a total of 6 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ______________________

1. (11 total points) Consider the following system of linear equations.

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ \qquad 2x_2 + 5x_3 = -2 \\ x_1 + 3x_2 + 6x_3 = -2 \end{cases}$$

(a) (9 points) Find all solutions to this system of linear equations.

(b) (2 points) Let A be the coefficient matrix corresponding to the system of equations. Are the columns of A linearly independent?

2. (11 total points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 5 & 8 \end{bmatrix}.$$

(a) (9 points) Find the inverse of A or state that A is not invertible.

(b) (2 points) Use your answer to part (a) to find the inverse of A^{T} or state that A^{T} is not invertible.

3. (11 total points) Let

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 1 & -1 & 4 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 5 \end{bmatrix}.$$

(a) (9 points) Find the determinant of A.

(b) (2 points) How many solutions are there to the equation $A\mathbf{x} = \mathbf{0}$?

4. (9 total points) For each statement, tell whether it is true or false and explain why. You do not need to prove anything, but your explanation should be clear.

   (a) (3 points) True or false: If A is a nonsingular $n \times n$ matrix and $A^2 = A$, then $A = \mathrm{I}_n$ (the identity matrix).

   (b) (3 points) True or false: If A and B are $n \times n$ symmetric matrices, then $A + B$ is also symmetric.

   (c) (3 points) True or false: If A and B are nonsingular $n \times n$ matrices, then $A + B$ is nonsingular.

5. (8 points) Prove that if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ of vectors in $\mathbf{R}^n$ is linearly dependent, then the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is also linearly dependent.