1. The augmented matrix is $\left[\begin{array}{cccc|c} 2 & -2 & 0 & 2 & 7 \\ 1 & -2 & -1 & -1 & 2 \\ 3 & -2 & 1 & 1 & 2 \end{array}\right]$, which in reduced row echelon form is $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & -5/2 \\ 0 & 1 & 1 & 0 & -7/2 \\ 0 & 0 & 0 & 1 & 5/2 \end{array}\right]$. So the general solution is $\begin{pmatrix} x_1 = -5/2 - x_3 & x_3 = x_3 \\ x_2 = -7/2 - x_3 & x_4 = 5/2 \end{pmatrix}$. The vector form is $\vec{\mathbf{x}} = \begin{bmatrix} -5/2 \\ -7/2 \\ 0 \\ 5/2 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} x_3$.

2. (a) $BA = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $B\vec{\mathbf{x}} = \begin{bmatrix} 12 \\ -7 \\ 0 \end{bmatrix}$.

   (b) Nonsingular because it row reduces to the identity.

   (c) No. If they were, they would have the same rref, but A row reduces to the identity, while B is already row reduced.

3. (a) $c \neq ab$.

   (b) Dependent

4. (a) $A^{-1} = \begin{bmatrix} 0 & 1/2 & -1/6 \\ 1/5 & -1 & 4/15 \\ 0 & 0 & 1/3 \end{bmatrix}$.

   (b) $A^T B^{-1} = B^{-1}(A^{-1})^T = \begin{bmatrix} 1/6 & -7/15 & 2/3 \\ -1/6 & 7/15 & 1/3 \\ 1/2 & -1 & 0 \end{bmatrix}$

5. (a) Since $A^T = (a_{ji})$ and $A^T = -A = (-a_{ij})$, we have $a_{ij} = -a_{ji}$. In particular, $a_{ii} = -a_{ii}$. But the only number equal to its negative is 0.

   (b) We need to show that $(A^2)^T = A^2$. We have

$$(A^2)^T = (AA)^T = A^T A^T = (A^T)^2 = (-A)^2 = A^2.$$