Print Your Name

Student ID #

Problem	Total Points	Score
1	8	
2	10	
3	8	
4	11	
5	8	
Total	45	

Directions

- Please check that your exam contains a total of 6 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: _____

1. (8 points) Find the vector form of the general solution to the following system of equations:

$$\begin{cases} 2x_1 - 2x_2 \qquad + 2x_4 = 7 \\ x_1 - 2x_2 - x_3 - x_4 = 2 \\ 3x_1 - 2x_2 + x_3 + x_4 = 2 \end{cases}$$

2. (10 total points) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and $\vec{\mathbf{x}} = \begin{bmatrix} 8 \\ -7 \\ 4 \end{bmatrix}$

   (a) (4 points) Compute BA and $B\vec{\mathbf{x}}$.

   (b) (3 points) Is A singular or nonsingular? Explain your answer.

   (c) (3 points) Are A and B row equivalent? Explain your answer.

3. (8 total points)

   (a) (4 points) Under what condition is the following set of vectors linearly independent? Express your answer in terms of a, b, and c.

$$\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} b \\ c \end{bmatrix} \right\}$$

   (b) (4 points) Is the following set of vectors linearly dependent or independent? Explain your answer.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \\ 5 \end{bmatrix} \right\}$$

4. (11 total points) Let $A = \begin{bmatrix} 10 & 5 & 1 \\ 2 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

   (a) (6 points) Find the inverse of A.

   (b) (5 points) Find $(A^T B)^{-1}$.

5. (8 total points) We say that a matrix A is **skew-symmetric** if $A^T = -A$.
   Recall that a matrix B is **symmetric** if $B^T = B$.
   Prove the following (relatively unrelated) facts:

   (a) (4 points) If $A = (a_{ij})$ is a skew-symmetric $n \times n$ matrix then every entry a_{ii} along the main diagonal is zero.

   (b) (4 points) If A is a skew-symmetric $n \times n$ matrix, then A^2 is symmetric.