1. (a) $\begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
   (b) The system is inconsistent.
   (c) $\begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} x_2$, because $x_3 = 0$, x_2 is free, and $x_1 = -1/2x_2$.
   (d) The reduced row echelon form of B, which is B itself, is not the same as the reduced row echelon form of A.

2. (a) We must solve the system corresponding to the augmented matrix $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & a & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$. This is row equivalent to $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & a-3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$. If $a = 3$ then the system has infinitely many solutions, so the vectors are linearly dependent.
   (b) Linearly independent.

3. (a) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1/3 & 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 \\ 1/2 \\ 2 \\ 0 \end{bmatrix}$
   (c) $\Delta = 3a$, so a must be nonzero.
   (d) $\frac{1}{3a}\begin{bmatrix} a & 0 \\ -8 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -\frac{8}{3a} & \frac{1}{a} \end{bmatrix}$

4. (a) False. Two matrices in RREF are row equivalent only if they are equal.
   (b) True.
   (c) False. For example the set $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is linearly dependent, but none of the vectors is a scalar multiple of another.
   (d) False. $(AB)^2 = ABAB$ which is not usually equal to $A^2B^2 = AABB$ because AB is not usually equal to BA.
   (e) False. The only possibilities for a system of linear equations are to have no solutions, one solution, or infinitely many solutions.
   (f) True.

5. (a) If we multiply B^{T} by B, we get I:

$$\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

   (b) We get $\|\vec{\mathbf{y}}\| = 5$, $B\vec{\mathbf{y}} = \begin{bmatrix} 24/5 \\ 7/5 \end{bmatrix}$, and $\|B\vec{\mathbf{y}}\| = 5$.
   (c) This is uses the definition of orthogonal and properties of the transpose:

$$\begin{aligned} \|A\vec{\mathbf{x}}\| &= \sqrt{(A\vec{\mathbf{x}})^{\mathrm{T}} A\vec{\mathbf{x}}} && \text{by definition of length} \\ &= \sqrt{\vec{\mathbf{x}}^{\mathrm{T}} A^{\mathrm{T}} A\vec{\mathbf{x}}} && \text{by property of transpose} \\ &= \sqrt{\vec{\mathbf{x}}^{\mathrm{T}} I\vec{\mathbf{x}}} && \text{because } A \text{ is orthogonal} \\ &= \sqrt{\vec{\mathbf{x}}^{\mathrm{T}}\vec{\mathbf{x}}} && \text{by property of identity} \\ &= \|\vec{\mathbf{x}}\| && \text{by definition of length.} \end{aligned}$$