Print Your Name

Student ID #

Problem	Total Points	Score
1	12	
2	7	
3	14	
4	7	
5	10	
Total	50	

Directions

- Please check that your exam contains a total of 6 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ______________________

1. (12 total points) Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 4 \\ 8 & 4 & 10 \end{bmatrix}$.

   (a) (5 points) Use row operations to convert A into reduced row echelon form.

   (b) (2 points) Suppose that A is the augmented matrix for a system of equations. Find all solutions to the system of equations.

   (c) (3 points) Find all vectors $\vec{\mathbf{x}}$ such that $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$. (Where $\vec{\mathbf{0}}$ is the zero vector in R^3).

   (d) (2 points) Write one sentence explaining why A is **not** row equivalent to $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

2. (7 total points) Let

$$\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \vec{\mathbf{v}}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \vec{\mathbf{v}}_3 = \begin{bmatrix} 1 \\ 2 \\ a \\ 0 \end{bmatrix}$$

(a) (4 points) Find a value a so that the set $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is linearly **dependent**.

(b) (3 points) Is the set $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2\}$ linearly independent or linearly dependent? (Show your work, of course)

3. (14 total points) Let

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \vec{\mathbf{b}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 \\ 8 & a \end{bmatrix}$$

(a) (6 points) Find the inverse of A.

(b) (2 points) Use your answer above to find a vector $\vec{\mathbf{x}}$ such that $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

(c) (3 points) For what values of a is the matrix B invertible?

(d) (3 points) For values of a in part (c), what is B^{-1}?

4. (7 points) True or False. No explanation is necessary. Scoring will be as follows:

Number correct	6	5	4	3	2	1	0
Score	7	5	3	1	0	0	0

(a) Any two matrices that are in reduced row echelon form are row equivalent.

(b) The $(n \times n)$ identity matrix is nonsingular for every n.

(c) If $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is a linearly dependent set, then one of the three vectors is a scalar multiple of another.

(d) If A and B are $(n \times n)$ matrices, then $(AB)^2 = A^2B^2$.

(e) There are systems of linear equations that have exactly two solutions.

(f) If a consistent system of equations has more variables than equations, then it must have infinitely many solutions.

5. (10 total points) An $(n \times n)$ matrix A is **orthogonal** if $A^{\mathrm{T}}A = I$. Remember that the length of a vector is defined $\|\vec{\mathbf{x}}\| = \sqrt{\vec{\mathbf{x}}^{\mathrm{T}}\vec{\mathbf{x}}}$.

   Let $B = \begin{bmatrix} 4/5 & 3/5 \\ -3/5 & 4/5 \end{bmatrix}$ and $\vec{\mathbf{y}} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

   (a) (3 points) Use the definition above to show that the matrix B is orthogonal.

   (b) (3 points) Compute $\|\vec{\mathbf{y}}\|$, $B\vec{\mathbf{y}}$, and $\|B\vec{\mathbf{y}}\|$.

   (c) (4 points) If A is any orthogonal $(n \times n)$ matrix and $\vec{\mathbf{x}}$ is any vector in R^n, show that $\|\vec{\mathbf{x}}\| = \|A\vec{\mathbf{x}}\|$.