Print your name: ______________________________

This exam has 9 questions on 5 pages, worth a total of 45 points.

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	6	
6	4	
7	5	
8	5	
9	5	
Total	45	

You should:

- write complete solutions or you may not receive credit.
- box your final answer.

You may:

- use one sheet of notes.
- write on the backs of the pages if you need more room.

Please do not:

- come to the front of the room to ask questions (raise your hand).
- use a calculator or any other unauthorized electronic device.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ______________________

1. (5 points) For exactly $1 at Trader Joe's, you can buy either (a) 1 apple, 1 banana, and 1 orange, (b) 2 bananas and 3 oranges, or (c) 1 apple and 2 oranges. How much for just one apple?

2. (5 points) For what value(s) of a is the following system of equations consistent?

$$\begin{aligned} x + 2y &= 1 \\ 2x + 4y &= a \end{aligned}$$

3. (5 points) Suppose $A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB + AC = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$.

   If possible, find $B + C$.

4. (5 points) Calculate the inverse of $D = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

5. (6 points) Consider the following systems of equations:

I. A 4×5 homogeneous system.
II. A 4×2 consistent system.
III. A 3×4 system.

(An $m \times n$ system of equations has m equations and n variables.)

Answer the following:

(a) Which of these WILL ALWAYS have at least one solution?
a) I only b) II only c) III only d) I and II only
e) I and III only f) II and III only g) I, II, and III h) None of these

(b) Which of these MAY have infinitely many solutions?
a) I only b) II only c) III only d) I and II only
e) I and III only f) II and III only g) I, II, and III h) None of these

(c) Which of these WILL ALWAYS have infinitely many solutions?
a) I only b) II only c) III only d) I and II only
e) I and III only f) II and III only g) I, II, and III h) None of these

6. (4 points) Answer each of the following

(a) Suppose A, B, and C are nonsingular matrices, and $C^{-1} = AB$. Which one of the following is equal to A^{-1}?
a) CB b) $B^{-1}C$ c) BC d) $C^{-1}B$

(b) True or false: If $AC = BC$, then $A = B$.

7. (5 points) Is the set of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ linearly dependent or independent?

8. (5 points) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a set of four vectors in $\mathbf{R}^3$. Prove that the set is linearly dependent. (Do not use the theorem that states a set of more than n vectors in $\mathbf{R}^n$ is linearly dependent.)

9. (5 points) Let N be a square matrix with $N^2 = \mathcal{O}$. Prove that N is singular. (Remember that N^2 means N times N, and $\mathcal{O}$ is the zero matrix.)