1. (a) Possible answers: $\left\{ \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix} \right\}$ or $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

   (b) Possible answer: $\left\{ \begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -18 \\ 13 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

   (c) Possible answer: $\left\{ \begin{bmatrix} 2 \\ 0 \\ 18 \\ 36 \\ 16 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 21 \\ 12 \\ 10 \end{bmatrix} \right\}$.

   (d) Rank = 2, Nullity = 3.

2. (a) $A = \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$.

   (b) 2

3. Using Gram-Schmidt with the vectors in this order, you get $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -4 \\ -2 \end{bmatrix} \right\}$.

4. Use least squares on $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 88 \\ 66 \\ 50 \end{bmatrix}$ to get $\mathbf{x}^* = \begin{bmatrix} 106 \\ -19 \end{bmatrix}$. Thus the answer is $P(t) = 106 - 19t$.

5. (a) **False.** $T(\vec{\mathbf{x}} + \vec{\mathbf{y}}) \neq T(\vec{\mathbf{x}}) + T(\vec{\mathbf{y}})$.

   (b) **True** (Theorem 17).

   (c) **True** (Theorem 13).

   (d) **False.** For example, U and V could be different lines in $\mathbb{R}^2$. Then they both have dimension 1, but U is not a subset of V.

   (e) **True.**

   (f) **True.** A basis is linearly independent, and a matrix is nonsingular if and only if its columns are linearly independent.

6. Let $W = U \cap V$. We must check that W satisfies the three properties of Theorem 2:

   (s1) Since U and V are subspaces, $\vec{\mathbf{0}}$ is in both U and V, so it is in W.

   (s2) Let $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ be vectors in W. Then $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are in both U and V. Since U is a subspace, $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ is in U. Since V is a subspace, $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ is also in V. So $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ is in W, which is what we needed to prove.

   (s3) Let a be any scalar and $\vec{\mathbf{x}}$ a vector in W. Then $\vec{\mathbf{x}}$ is in both U and V. Since U and V are subspaces, $a\vec{\mathbf{x}}$ is in both U and V. So $a\vec{\mathbf{x}}$ is in W.

   Therefore, W is a subspace of $\mathbb{R}^n$.