Print Your Name

Student ID #

Problem	Total Points	Score
1	12	
2	7	
3	7	
4	7	
5	11	
6	6	
Total	50	

Directions

- Please check that your exam contains a total of 7 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ____________________

1. (12 total points) Let

$$A = \begin{bmatrix} 2 & 0 & 18 & 36 & 16 \\ 5 & 6 & 21 & 12 & 10 \\ -1 & 6 & -33 & -96 & -38 \end{bmatrix}$$

To save you time, I have done the following helpful calculations:

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 9 & 18 & 8 \\ 0 & 1 & -4 & -13 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad A^{\mathrm{T}} = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 6 & 6 \\ 18 & 21 & -33 \\ 36 & 12 & -96 \\ 16 & 10 & -38 \end{bmatrix} \qquad \text{rref}(A^{\mathrm{T}}) = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) (3 points) Find a basis for the range of A.

(b) (3 points) Find a basis for the nullspace of A.

(c) (3 points) Find a basis for the row space of A such that each vector in the basis is one of the rows of A.

(d) (3 points) What are the rank and nullity of A?

2. (7 total points) Define a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ as follows:

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} 2b \\ -3a \end{bmatrix}$$

(a) (5 points) Find the matrix A so that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$.

(b) (2 points) What is the dimension of the range of T?

3. (7 points) Let W be a subspace of $\mathbb{R}^4$, and suppose that $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is a basis for W, where

$$\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{\mathbf{v}}_3 = \begin{bmatrix} 4 \\ 8 \\ -4 \\ 4 \end{bmatrix}.$$

Find an orthogonal basis for W.

4. (7 points) Suppose that we have the following data for a certain population:

Time (years)	1	2	3
Population	88	66	50

Use the least-squares method to find a linear fit for this data. In other words, find a function $P(t) = a + bt$ that best approximates this population data.

5. (11 points) True or false. No justification is necessary.

   (a) The function defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 - 9 \end{bmatrix}$ is a linear transformation.

   (b) For any $(m \times n)$ matrix A and $(m \times 1)$ vector $\vec{\mathbf{b}}$, the equation $A^{\mathrm{T}} A\vec{\mathbf{x}} = A^{\mathrm{T}}\vec{\mathbf{b}}$ has at least one solution.

   (c) Any orthogonal set of nonzero vectors in $\mathbb{R}^n$ is linearly independent.

   (d) If U and V are subspaces of $\mathbb{R}^n$ and $\dim U \leq \dim V$, then U is a subspace of V (that is, every vector of U is also in V).

   (e) The function from $\mathbb{R}^n$ to $\mathbb{R}^n$ defined by $T(\vec{\mathbf{x}}) = \vec{\mathbf{0}}$ is a linear transformation.

   (f) If $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2 \ldots, \vec{\mathbf{v}}_n\}$ is a basis for $\mathbb{R}^n$, then the matrix $A = [\vec{\mathbf{v}}_1 \ \vec{\mathbf{v}}_2 \ \cdots \ \vec{\mathbf{v}}_n]$ whose columns are these basis vectors is a nonsingular matrix.

6. (6 points) Let U and V be subspaces of $\mathbb{R}^n$. Recall that the *intersection* of U and V is defined as follows:

$$U \cap V = \{\vec{\mathbf{x}} : \vec{\mathbf{x}} \text{ is in } U \text{ and } \vec{\mathbf{x}} \text{ is in } V\}.$$

Prove that $U \cap V$ is a subspace of $\mathbb{R}^n$.