On problems involving bases, there will be more than one correct response.

1. (a) and (b) are the same subspace. Two possible bases are

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 9 \\ 1 \end{bmatrix} \right\} \text{ and } \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(c) $\left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ -3 \\ 1 \end{bmatrix} \right\}$. (d) A has rank 3 and nullity 2. B has rank 3 and nullity 1.

2. (a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \right\}$

3. (a) $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ (b) 3,4,5 or 6 (c) 4×5

   (d) $\vec{\mathbf{v}}_4$ together with any one of the other three.

4. (a) Suppose that $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$ are in W. Then the sum is $\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \\ x_4+y_4 \end{bmatrix}$. Since $x_2+x_3+x_4=0$ and $y_2+y_3+y_4=0$, then $(x_2+y_2)+(x_3+y_3)+(x_4+y_4)=0$. So the sum is in W, which means that W is closed under addition.

   (b) Since W is defined by a homogeneous system of equations, it is the null space of $\begin{bmatrix} 0 & 1 & 1 & 2 \end{bmatrix}$. A basis is $\{[1\ 0\ 0\ 0]^T, [0\ -1\ 1\ 0]^T, [0\ -1\ 0\ 1]^T\}$.

5. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 0 \\ 3 \\ 6 \\ 10 \end{bmatrix}$. $A^TA = \begin{bmatrix} 4 & 5 \\ 5 & 15 \end{bmatrix}$ and $A^T\vec{\mathbf{b}} = \begin{bmatrix} 19 \\ 45 \end{bmatrix}$. The solution to the system of equations is $a = 12/7$ and $b = 17/7$, so the equation of the line is $y = \frac{12}{7} + \frac{17}{7}y$.

6. (a) $A\vec{\mathbf{v}}_1 = \vec{\mathbf{0}}$ and $A\vec{\mathbf{v}}_2 = \vec{\mathbf{0}}$.

   (b) By row reducing A, we see that the dimension of $\mathcal{N}(A)$ is 2. Since $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are are linearly independent, and any two linearly independent vectors in a 2-dimensional space form a basis, this set is a basis for $\mathcal{N}(A)$.