Print Your Name

Student ID #

Problem	Total Points	Score
1	8	
2	6	
3	12	
4	8	
5	6	
6	6	
Total	46	

Directions

- Please check that your exam contains a total of 7 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ______________________

1. (8 points) Suppose that the following two matrices row reduce as follows:

$$A = \begin{bmatrix} 1 & -4 & 2 & 0 & -1 \\ -2 & 8 & -4 & 0 & 2 \\ -3 & 12 & 6 & 9 & 6 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -4 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -2 & -3 & 0 \\ -4 & 8 & 12 & 0 \\ 2 & -4 & 6 & 1 \\ 0 & 0 & 9 & 1 \\ -1 & 2 & 6 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for Span $\left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 8 \\ 12 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 9 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 6 \\ 1 \end{bmatrix} \right\}$.

(b) Find a basis for the row space of B.

(c) Find a basis for the null space of A.

(d) Fill in the following table.

	rank	nullity
A		
B		

2. (6 points) Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$.

   (a) Find an orthogonal basis for W.

   (b) Now find an orthonormal basis for W.

3. (12 points)

(a) If T is a linear transformation and $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, what is $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$?

(b) Suppose that W is a subspace of R^7 that is spanned by $\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3, \vec{\mathbf{w}}_4, \vec{\mathbf{w}}_5, \vec{\mathbf{w}}_6\}$ and $\{\vec{\mathbf{w}}_1, \vec{\mathbf{w}}_2, \vec{\mathbf{w}}_3\}$ is a linearly independent set. What are all the possibilities for the dimension of W?

(c) Suppose that the nullity of A is 2, the rank of A is 3, and the nullity of A^T is 1. What are the dimensions of A? (Express your answer in the form rows $\times$ columns.)

(d) Suppose that V is a 2-dimensional subspace of R^n and $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3, \vec{\mathbf{v}}_4$ are four vectors that satisfy $\vec{\mathbf{v}}_1 = 10\vec{\mathbf{v}}_3$ and $\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3 = 0$. Find a basis for V.

4. (8 points) Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : x_2 + x_3 + x_4 = 0 \right\}$. This is a subspace of R^4.

   (a) Show that W is closed under addition (i.e. satisfies property (s2) of Theorem 2).

   (b) Find a basis for W.

5. (6 points) Find a least squares linear fit for the following data. In other words, find an equation $y = a + bx$ that best approximates the data.

x	-1	1	2	3
y	0	3	6	10

6. (6 points) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$, $\vec{\mathbf{v}}_1 = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$, and $\vec{\mathbf{v}}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

   (a) Show that $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are in $\mathcal{N}(A)$.

   (b) Prove that $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2\}$ is a basis for $\mathcal{N}(A)$.