1. (a) Two of the many possibilities: $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}; \left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

   (b) One possibility: $\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

   (c) See above

   (d) 2

2. (a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

   (b) $\frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}$.

   (c) 3

3. (a) $T\left(\begin{array}{c} a \\ b \end{array} \right) = \begin{bmatrix} 0 \\ \frac{1}{2}a + \frac{1}{3}b \\ a + \frac{2}{3}b \end{bmatrix}$.

   (b) $A = \begin{bmatrix} 0 & 0 \\ 1/2 & 1/3 \\ 1 & 2/3 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$, $\vec{\mathbf{b}} = \begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$. $A^{\mathrm{T}}A = \begin{bmatrix} 3 & 3 \\ 3 & 5 \end{bmatrix}$, $A^{\mathrm{T}}\vec{\mathbf{b}} = \begin{bmatrix} 13 \\ 21 \end{bmatrix}$. $\vec{\mathbf{x}} = \begin{bmatrix} 1/3 \\ 4 \end{bmatrix}$.

   So the linear fit equation is $y = 1/3 + 4x$.

5. (a) For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

   (b) 2

   (c) No because it does not span R^3.

6. Let $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ be in R^2 and let a be a scalar. Then $T(\vec{\mathbf{x}}) + T(\vec{\mathbf{x}}) = \vec{\mathbf{0}} + \vec{\mathbf{0}} = \vec{\mathbf{0}}$, and $T(\vec{\mathbf{x}} + \vec{\mathbf{y}}) = \vec{\mathbf{0}}$, so they are the same. Also, $T(a\vec{\mathbf{x}}) = \vec{\mathbf{0}}$ and $aT(\vec{\mathbf{x}}) = a\vec{\mathbf{0}} = \vec{\mathbf{0}}$, so they are the same. Therefore T is a linear transformation.