Print Your Name	Student ID $\#$							

Problem	Total Points	Score
1	12	
2	11	
3	7	
4	7	
5	8	
6	5	
Total	50	

Directions

- Please check that your exam contains a total of 7 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: _____

- 1. (12 total points) Let $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 3 & 3 & 6 & 6 \\ 1 & 2 & 2 & 4 \end{bmatrix}$.
 - (a) (4 points) Find a basis for the range of A.

(b) (4 points) Find a basis for the nullspace of A.

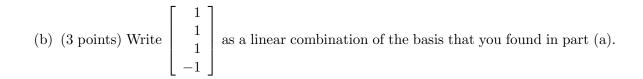
(c) (2 points) Find a basis for the column space of A that is different from the basis you found in part (a).

(d) (2 points) What is the rank of A?

2. (11 total points) Let W be the subspace of \mathbb{R}^4 that has the following basis:

$$\left\{ \begin{bmatrix} 1\\1\\1\\1\\1 \end{bmatrix}, \begin{bmatrix} 1\\1\\-3\\-3 \end{bmatrix}, \begin{bmatrix} 1\\2\\3\\4 \end{bmatrix} \right\}.$$

(a) (6 points) Find an orthogonal basis for W.



(c) (2 points) What is the dimension of W?

Winter 2010

3. (7 points) Suppose that T is a linear transformation from \mathbb{R}^2 to \mathbb{R}^3 and you know that

$$T\left(\left[\begin{array}{c}2\\0\end{array}\right]\right) = \left[\begin{array}{c}0\\1\\2\end{array}\right] \qquad \qquad T\left(\left[\begin{array}{c}0\\3\end{array}\right]\right) = \left[\begin{array}{c}0\\1\\2\end{array}\right].$$
(a) What is $T\left(\left[\begin{array}{c}a\\b\end{array}\right]\right)$?

(b) Find a matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ for all $\vec{\mathbf{x}}$ in R^2 .

4. (7 points) Use the least squares method to find a linear fit for the following data:

(In other words, find an equation y = a + bx that best approximates the data.)

- 5. (8 total points)
 - (a) (3 points) Suppose that V is a subspace of R^2 , and $\begin{bmatrix} 1\\ 0 \end{bmatrix} \in V$. Find two other vectors which must also be in V.

(b) (2 points) What is the nullity of the matrix [2 24 2010]?

(c) (3 points) Is
$$\left\{ \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \begin{bmatrix} 0\\1\\0 \end{bmatrix} \right\}$$
 a basis for R^3 ? Why or why not?

6. (5 points) Define a function $T: \mathbb{R}^2 \to \mathbb{R}^2$ as follows:

$$T\left(\left[\begin{array}{c} x_1\\ x_2 \end{array}\right]\right) = \left[\begin{array}{c} 0\\ 0 \end{array}\right].$$

Prove that T is a linear transformation.