Print Your Name

Student ID #

Problem	Total Points	Score
1	12	
2	11	
3	7	
4	7	
5	8	
6	5	
Total	50	

Directions

- Please check that your exam contains a total of 7 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ____________________

1. (12 total points) Let $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 3 & 3 & 6 & 6 \\ 1 & 2 & 2 & 4 \end{bmatrix}$.

   (a) (4 points) Find a basis for the range of A.

   (b) (4 points) Find a basis for the nullspace of A.

   (c) (2 points) Find a basis for the column space of A that is different from the basis you found in part (a).

   (d) (2 points) What is the rank of A?

2. (11 total points) Let W be the subspace of R^4 that has the following basis:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}.$$

(a) (6 points) Find an orthogonal basis for W.

(b) (3 points) Write $\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ as a linear combination of the basis that you found in part (a).

(c) (2 points) What is the dimension of W?

3. (7 points) Suppose that T is a linear transformation from R^2 to R^3 and you know that

$$T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \qquad\qquad T\left(\begin{bmatrix} 0 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

(a) What is $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right)$?

(b) Find a matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ for all $\vec{\mathbf{x}}$ in R^2.

4. (7 points) Use the least squares method to find a linear fit for the following data:

x	0	1	2
y	1	3	9

(In other words, find an equation $y = a + bx$ that best approximates the data.)

5. (8 total points)

(a) (3 points) Suppose that V is a subspace of R^2, and $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \in V$. Find two other vectors which must also be in V.

(b) (2 points) What is the nullity of the matrix $[2 \quad 24 \quad 2010]$?

(c) (3 points) Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ a basis for R^3? Why or why not?

6. (5 points) Define a function $T : R^2 \to R^2$ as follows:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Prove that T is a linear transformation.