Print your name: ____________________

This exam has 7 questions on 5 pages, worth a total of 45 points.

Page	Points	Score
1	10	
2	5	
3	15	
4	5	
5	10	
Total	45	

You should:

- write complete solutions or you may not receive credit.
- $\boxed{\text{box your final answer.}}$

You may:

- use one sheet of notes.
- write on the backs of the pages if you need more room.

Please do not:

- come to the front of the room to ask questions (raise your hand).
- use a calculator or any other unauthorized electronic device.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ____________________

1. (10 points) Let $A = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 1 & 1 & -1 & -2 \\ 3 & 1 & 3 & 2 \\ 2 & 2 & -2 & -4 \end{bmatrix}$.

(a) Find a basis for the row space of A.

(b) Find a basis for the null space of A.

(c) What is the rank of A?

(d) What is the nullity of A?

2. (5 points) Find an orthogonal basis for the 3-dimensional subspace of $\mathbf{R}^4$ spanned by the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 0 \\ 1 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 1 \\ -3 \\ -1 \end{bmatrix}$.

3. (9 points) For each matrix described below, determine the dimension of the null space and the dimension of the range.

   (a) $A = \begin{bmatrix} 1 & -1 & 3 & 5 & -2 & 10 & -2 \\ 0 & 1 & -3 & 0 & 1 & 9 & 3 \end{bmatrix}.$

   Dimension of null space of A: ________ Dimension of range of A: ________

   (b) B is the 3×3 matrix such that $B\mathbf{x}$ is the reflection of $\mathbf{x}$ across the plane $x + y = 0$.

   Dimension of null space of B: ________ Dimension of range of B: ________

   (c) C is the 2×3 matrix such that $C \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ z \end{bmatrix}.$

   Dimension of null space of C: ________ Dimension of range of C: ________

4. (6 points) Let C be the matrix from question 3(c).

   (a) Find a basis for the null space of C.

   (b) Find a basis for the range of C.

5. (5 points) Use the least squares method to find a linear function that best approximates the following data:

x	-2	-1	0	1	2
y	2	1	-1	0	-1

6. (5 points) Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y + z = 0 \right\}$.

   Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be two nonzero vectors in W with $\mathbf{v}_1^T \mathbf{v}_2 = 0$. Prove that $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for W.

7. (5 points) Let $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0 \text{ and } y \geq 0 \right\}$.

   For each of the following, answer yes or no. No explanation is necessary.

   (a) Is the zero vector $\mathbf{0}$ in S?

   (b) If $\mathbf{v}_1$ and $\mathbf{v}_2$ are in S, is $\mathbf{v}_1 + \mathbf{v}_2$ in S?

   (c) If $\mathbf{v}$ is in S and a is a real number, is $a\mathbf{v}$ in S?