1. (a) $A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} c & d \\ a & b \end{bmatrix}$

   (c) 2

   (d) The characteristic polynomial is $\lambda^2 - 1$, so the eigenvalues are 1 and -1.
   The eigenspace E_1 is the nullspace of the matrix $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, and a basis is $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.
   The eigenspace E_{-1} is the nullspace of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, and a basis is $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$.

   (e) Yes. An eigenbasis is $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$.

   (f) The line is $y = x$ because vectors along this line (multiples of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$) are not changed and vectors perpendicular to this line (multiples of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$) are flipped.

2. (a) The characteristic polynomial is $\begin{vmatrix} -\lambda & -2 & -2 \\ 0 & 1-\lambda & -1 \\ 0 & 1 & 3-\lambda \end{vmatrix} = -\lambda^3 + 4\lambda^2 - 4\lambda = -\lambda(\lambda-2)^2$.

   (b) A has an eigenvalue of 0 with algebraic multiplicity 1 and an eigenvalue of 2 with algebraic multiplicity 2.

   (c) The geometric multiplicity must be between 1 and the algebraic multiplicity. So the geometric multiplicity of 0 must be 1. The geometric multiplicity of 2 must also be 1, because if it were 2 then the matrix would be diagonalizable.

3. (a) E_2 is the nullspace of the matrix $\begin{pmatrix} -5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. A basis is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

   (b) Yes. The geometric multiplicity of 2 is 2, but the algebraic multiplicity is 3.

4. (a) $A = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

5. (a) For example, $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. Any linearly independent set with one or two vectors will work. If you put 3 linearly independent vectors then you would have a basis.

   (b) The subspace with basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ (This is a line through the origin).

   (c) The subspace with basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ (This is a plane through the origin).

(d) For example, a 2×3 matrix and a 3×2 matrix.

(e) Solve the system of equations $\left(\begin{array}{cc|c} 1 & 2 & 9 \\ 1 & 3 & 14 \end{array}\right)$ and get $x_1 = -1, x_2 = 5$. So we have $\begin{bmatrix} 9 \\ 14 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 5 \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

6. (a) False. It doesn't include the zero vector.
   (b) False. This is only true for matrices that are not defective.
   (c) True. There is always at least one solution, the trivial solution.
   (d) True. This is part of the definition of a basis.
   (e) True. A matrix is nonsingular if and only if it is invertible.

7. True. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and let a be any scalar. Then

$$\begin{aligned} T(\mathbf{x}+\mathbf{y}) &= T\left(\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \end{bmatrix}\right) = \begin{bmatrix} x_1+y_1 \\ x_1+y_1 \\ x_1+y_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} y+1 \\ y+1 \\ y+1 \end{bmatrix} \\ &= T(\mathbf{x}) + T(\mathbf{y}), \text{ and} \\ T(a\mathbf{x}) &= T\left(\begin{bmatrix} ax_1 \\ ax_2 \end{bmatrix}\right) = \begin{bmatrix} ax_1 \\ ax_1 \\ ax_1 \end{bmatrix} = a \begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix} = aT(\mathbf{x}). \end{aligned}$$

This is the definition of a linear transformation, so T is a linear transformation.

8. (a) $A^3 = AA^2 = AI = A$.
   (b)

$$\begin{aligned} (A+I)^2 &= (A+I)(A+I) \\ &= A^2 + IA + AI + I^2 \\ &= I + A + A + I \\ &= 2(A+I) \end{aligned}$$

   (c) A is an invertible matrix, with $A^{-1} = A$. So $A\mathbf{x} = \mathbf{b}$ is always consistent. In fact $\mathbf{x} = A\mathbf{b}$ is a solution, since $A(A\mathbf{b}) = A^2\mathbf{b} = I\mathbf{b} = \mathbf{b}$.
   (d) A is invertible, so its rank is n.
   (e) A is invertible so its nullity is 0.