Print Your Name

Student ID #

Problem	Total Points	Score
1	18	
2	11	
3	7	
4	8	
5	14	
6	6	
7	6	
8	10	
Total	80	

Directions

- Please check that your exam contains a total of 9 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ____________________

1. (18 total points) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

   (a) (3 points) Find A^{-1}.

   (b) (2 points) Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Calculate AB.

   (c) (2 points) What is the rank of A?

(continuing Problem 1, A is still $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$)

(d) (7 points) Find all eigenvalues of A, and for each eigenvalue, find a basis for the corresponding eigenspace.

(e) (3 points) Is A diagonalizable? Find an eigenbasis, if one exists.

(f) (1 point) The linear transformation $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ flips vectors in $\mathbb{R}^2$ across a line. Which line? [Hint: use your answer to part (d)]

2. (11 total points)

   Let $A = \begin{bmatrix} 0 & -2 & -2 \\ 0 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}$.

   (a) (6 points) Find the characteristic polynomial of A

   (b) (3 points) List the eigenvalues of A and their algebraic multiplicities. DO NOT FIND THE EIGENVECTORS.

   (c) (2 points) A is a defective (not diagonalizable) matrix. Use this information to tell me the geometric multiplicity of each eigenvalue.

3. (7 total points) Let $A = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$.

   The characteristic polynomial of A is $(\lambda + 3)(\lambda - 2)^3$.

   (a) (5 points) Find a basis for the eigenspace of A corresponding to eigenvalue $\lambda = 2$.

   (b) (2 points) Is A defective?

4. (8 total points) Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation, and suppose that

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad\qquad T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(a) (6 points) Find the matrix A such that $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$.

(b) (2 points) Use your answer above to find $T\left(\begin{bmatrix} 2/3 \\ 5/3 \end{bmatrix}\right)$

5. (14 total points)

   (a) (3 points) Give an example of a linearly independent set in $\mathbb{R}^3$ that is not a basis for $\mathbb{R}^3$.

   (b) (3 points) Give a specific example of a one-dimensional subspace of $\mathbb{R}^3$.

   (c) (3 points) Give a specific example of a two-dimensional subspace of $\mathbb{R}^3$.

   (d) (2 points) Give an example of two non-square matrices whose product is a square matrix.

   (e) (3 points) Write $\begin{bmatrix} 9 \\ 14 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

6. (6 points) True or false. No justification is necessary. No penalty for guessing.

   (a) The set $\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : 2x_1 - 3x_2 = 1 \right\}$ is a subspace of $\mathbb{R}^2$.

   (b) For any $n \times n$ matrix A, the sum of the dimensions of its eigenspaces is n.

   (c) Every homogeneous system of equations is consistent.

   (d) Every basis is linearly independent.

   (e) Every nonsingular matrix has an inverse.

7. (6 points) Define a function $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix}$.

   (a) True or false: T is a linear transformation.

   (b) Prove your answer to part (a).

8. (10 points) Let A be an $n \times n$ matrix such that $A^2 = I$ (Where I is the $n \times n$ identity matrix.)

   (a) (2 points) Prove that $A^3 = A$.

   (b) (2 points) Prove that $(A + I)^2 = 2(A + I)$.

   (c) (3 points) Prove that $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is consistent for any $\vec{\mathbf{b}} \in \mathbb{R}^n$.

   (d) (2 points) What is the rank of A?

   (e) (1 point) What is the nullity of A?