Print Your Name

Student ID #

Problem	Total Points	Score
1	9	
2	12	
3	11	
4	9	
5	11	
6	14	
7	9	
Total	75	

Directions

- Please check that your exam contains a total of 9 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ____________________

1. (9 total points)

   (a) (4 points) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$ where a and b are nonzero. Find A^{-1}.

   (b) (3 points) Is the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ linearly independent? <u>Explain.</u>

   (c) (2 points) Give an example of a linearly independent set in R^5 that is not a basis for R^5.

2. (12 total points) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) (9 points) Find all eigenvalues of A, and for each eigenvalue find a basis for the corresponding eigenspace.

(b) (3 points) Is A defective? Explain why or why not.

3. (11 total points) Let $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and suppose that B is a 4×4 matrix with $\det B = 5$.

   (a) (5 points) Find $\det A$.

   (b) (2 points) What is $\det(AB)$?

   (c) (2 points) What is $\det(B^{-1})$?

   (d) (2 points) True or false: A has an inverse

4. (9 total points) Consider the following system of equations:

$$\begin{aligned} x + y &= 0 \\ 2x + 3y &= 0 \\ 4x + 5y &= 1 \end{aligned}$$

(a) (7 points) Find a least squares solution $\vec{\mathbf{x}}^*$ to the system of equations.

(b) (2 points) Calculate the error $\|A\vec{\mathbf{x}}^* - \vec{\mathbf{b}}\|$, where $[A \mid \vec{\mathbf{b}}]$ is the augmented matrix for the system of equations. Your answer should be a number.

5. (11 points) Let $A = \begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}$ and $\vec{\mathbf{x}} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$. Calculate $A^{25}\vec{\mathbf{x}}$.

6. (14 total points)

   (a) (3 points) Find a basis for the row space of $\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 3 & -3 \\ 1 & 1 \end{bmatrix}$.

   (b) (3 points) If A is a 3×17 matrix and $\text{rank}(A) = 2$, what is the nullity of A^T? (note the transpose)

   (c) (3 points) Let V be the nullspace of $\begin{bmatrix} 1 & -1 & 1 \\ 3 & 1 & -9 \end{bmatrix}$. Is $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ in V? Explain.

(d) (2 points) Give an example of a rank 2 matrix.

(e) (3 points) If B is a 4×4 matrix that has an eigenvalue of 2 with algebraic multiplicity 4 and geometric multiplicity 4, what is B?

7. (9 total points) Let A be an $n \times n$ matrix such that $A^3 = \mathcal{O}$. Note that A may not be the zero matrix.

   (a) (3 points) Prove that A cannot be invertible and is therefore singular.

   (b) (3 points) Prove that 0 is an eigenvalue of A (i.e. there is some $\vec{\mathbf{x}}$ such that $A\vec{\mathbf{x}} = 0\vec{\mathbf{x}}$).

   (c) (3 points) Prove that 0 is the only eigenvalue of A (i.e. if $A\vec{\mathbf{x}} = \lambda\vec{\mathbf{x}}$ then $\lambda = 0$).